Information hiding and retrieval in Rydberg wave packets using half-cycle pulses

J. M. Murray[1], S. N. Pisharody[1], H. Wen[1], C. Rangan[2], P. H. Bucksbaum[1]
[1] Department of Physics, University of Michigan, Ann Arbor, MI 48109-1120 and
[2] Department of Physics, University of Windsor, ON N9B 3P4, Canada.

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We demonstrate an information hiding and retrieval scheme with the relative phases between states in a Rydberg wave packet acting as the bits of a data register. We use a terahertz half-cycle pulse (HCP) to transfer phase-encoded information from an optically accessible angular momentum manifold to another manifold which is not directly accessed by our laser pulses, effectively hiding the information from our optical interferometric measurement techniques. A subsequent HCP acting on these wave packets reintroduces the information back into the optically accessible data register manifold which can then be 'read' out.

Coherent excited states of multilevel quantum systems have been proposed as quantum bit registers for storing and manipulating information [1, 2, 3]. Previous work has demonstrated the use of the quantum phases of the states in a Rydberg wave packet for information storage and showed the applicability of terahertz HCP's to retrieve and manipulate this information [4, 5, 6, 7, 8, 9]. Recently, we have measured the effects of an HCP on the phases and amplitudes of Rydberg p-state data registers[10]. In the current work, we extend this investigation to the possibility of performing sequential coherent operations on the stored information by using multiple HCP's. Since any program is a sequence of operations, the ability to perform multiple operations on all or selected parts of the stored data is an essential requirement for information processing.

We use a pair of HCP's acting on a Rydberg wave packet to demonstrate the storage and retrieval of information from the wave packet. In our experiments, information is stored in the phases of each of the states of the wave packet, with respect to the phase of a reference state. The first HCP acts on the wave packet storing the information, redistributing the complex probability amplitudes of the states in a deterministic manner[10]. Our means of detecting the stored information is a wave packet holography technique that has previously been applied to wave packet sculpting[11]. This technique uses interference with an $\ell = 1$ reference Rydberg wave packet, so it is sensitive only to the p-state populations in the wave packet. The non-p states populated by the HCP are therefore hidden from measurement. A subsequent HCP can redistribute the $\ell \neq 1$ state populations back into the p-states and make the information available for measurement. This ability to hide selective parts of the information and retrieve it at will allows us to introduce operators timed to occur between the two HCP's enabling us to act on a subset of the stored information. Here, we report the demonstration of this information hiding and retrieval scheme.

A tightly focused 1079 nm pulse from a Ti:Sapphire-pumped optical parametric amplifier excites ground state cesium atoms from an effusive source from the $6s$ state into an intermediate $7s$ launch state. A spectrally shaped 800 nm pulse excites an $n = 27, \ldots, 32$ p-state Rydberg wave packet with equal phases and approximately equal amplitudes[6]. This wave packet is subsequently kicked by a weak THz HCP polarized along the same direction as the laser pulses. The duration of the HCP (0.5ps) is significantly shorter than the Kepler period ($\sim$ 3ps). This suggests that an impulse approximation can be used, and theory and experiment confirm this[5]. The HCP impulsively transfers a momentum $Q = 0.0017$ a.u. (atomic units) to the Rydberg electron. This momentum kick transfers population from an initial $\ell = 1$ manifold into other p-states as well as $\ell \neq 1$ angular momentum states. With the correct choice of HCP delay T_1, we can selectively depopulate one of the states[10]. A second HCP is applied at time T_2, transferring amplitude from non-p states back into the $\ell = 1$ manifold. The resultant changes in phase for those p-states are then measured by exciting the reference p-state Rydberg wave packet at different delays, τ, with a second ultrafast pulse, identical to the first. The state selective field ionization (SSFI) spectrum is used to analyze the interference between the wave packets and determine the phase relationships between the states in the wave packet. Details of this phase measurement procedure have been described previously[10].

Information stored in the phases of the states of the wave packet is retrieved through correlation measurements. For two p-states $|j\rangle$ and $|k\rangle$, the noise-free correlation between their populations is given by

$$\begin{aligned} r_{jk}(\tau) &= \cos((\phi_{j1} - \phi_{k1}) - (\phi_{j2} - \phi_{k2}) - (\omega_j - \omega_k)\tau) \\ &= \cos(\Phi_{jk} - \omega_{jk}\tau). \end{aligned} \tag{1}$$

Here, ϕ_{j1} is the phase with which the state $|j\rangle$ is excited, a time τ before the reference pulse excites the j-component of the reference wave packet with phase ϕ_{j2}. The Rydberg state frequency for state $|j\rangle$ is denoted by ω_j. The amplitude of this correlation curve is unity in the absence of technical noise and decoherence. The presence of decoherence and background noise result in a measured

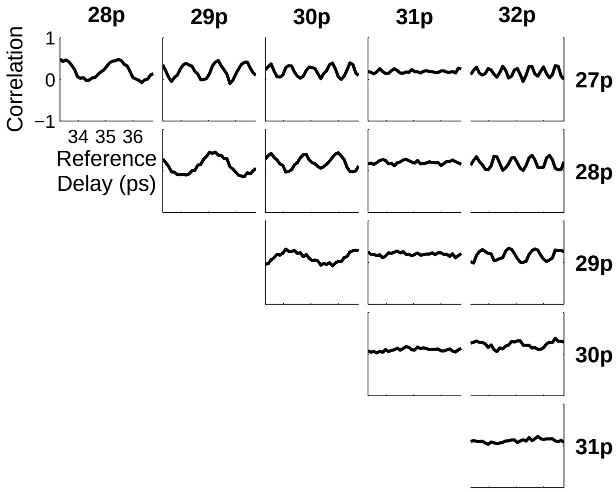

FIG. 1: Correlations vanish
The choice of HCP_1 delay $T_1 = 5$ ps moves population from the $31p$ state into neighboring angular momentum states such that the phase information that was contained in $31p$ cannot be measured. All correlations involving $31p$ vanish.

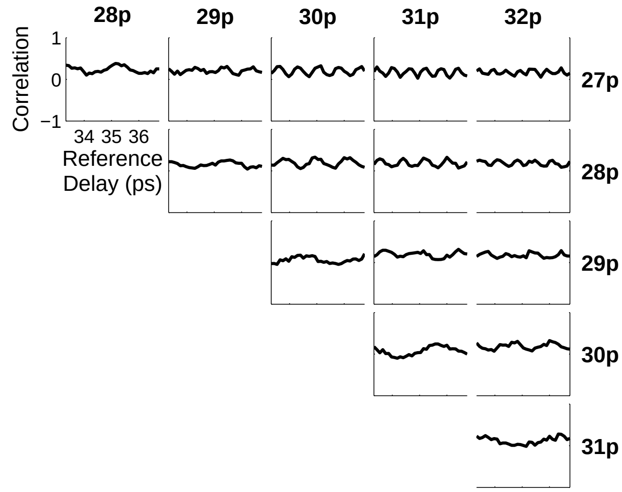

FIG. 2: Correlations recovered
Applying a second HCP at $T_2 = 6.3$ ps coherently transfers population from neighboring angular momentum states into the data register state, $n = 31p$. The correlations with $31p$ are seen to return; phase information is recovered.

correlation amplitude[10]

$$r_{jk}^{meas} = \sqrt{\left(1 - \frac{\sigma_{N_j}^2}{\sigma_{j_{meas}}^2}\right)\left(1 - \frac{\sigma_{N_k}^2}{\sigma_{k_{meas}}^2}\right)} \cdot r_{jk}, \qquad (2)$$

where σ_{N_j} is the standard deviation of the noise present in the measurement of state $|j\rangle$ and $\sigma_{j_{meas}}$ is the standard deviation of the population identified as state $|j\rangle$. Noise can also introduce an uncertainty $\Delta\Phi_{jk}$ in the phase of r_{jk}.

The correlation amplitude r_{jk} is a periodic function of the reference time delay, τ (Eq.1). The measured correlation curve is described primarily by its phase and its amplitude. The effect of a HCP is to modify the phases and amplitudes of the correlation curves. The amplitudes and phase shifts of the correlations due to the double HCP kick depend on the relative phases between the p-states, and hence on the HCP delays T_1 and T_2.

For information processing purposes, it is useful to quantify the amount of information that can be reliably retrieved, both with and without an HCP kick. When storing information in the quantum phase, ϕ_k, of a state $|k\rangle$ (with respect to some reference), one can divide the phase range, $[0,\ 2\pi)$ into N different partitions, each spanning a phase of $2\pi/N$ and representing a different discrete logical level[12]. For example, if we have 10 partitions, the scheme would be a decimal system. The information capacity of any digital encoding scheme with N logical levels scales as $\log N$[13, 14, 15]. The number of logical levels distinguishable in the quantum phase is related to the precision with which the phase can be measured, $\phi_k \pm \frac{\Delta\phi_k}{2}$. If the uncertainty in phase, $\Delta\phi_k$, is zero, the phase (and the encoded information) is known exactly; if the uncertainty is 2π, nothing is known about the phase. The information capacity of a state with phase uncertainty, $\Delta\phi_k$ can be written as

$$i_k = \log \frac{2\pi}{\Delta\phi_k}. \qquad (3)$$

The state phases, ϕ_k containing the encoded information are extracted using the correlation technique. We measure the correlation between all pairs of states. The phase associated with each state can be calculated as the difference in phases from multiple correlation measurements (for example, $\phi_k = \Phi_{k,\mathrm{ref}}$, and $\phi_k = \Phi_{k,j} - \Phi_{j,\mathrm{ref}}$ for all j). The noise level in these measurements influences the phase uncertainty associated with each phase difference. The quantity $\Delta\phi_k$ is measured as a weighted average of the phase uncertainties in the measured correlation curves,

$$\Delta\phi_k = \frac{\sum_j W_{jk}\Delta\Phi_{jk}}{\sum_j W_{jk}} = \frac{N-1}{\sum_j 1/\Delta\Phi_{jk}}, \qquad (4)$$

where $W_{jk} = 1/\Delta\Phi_{jk}$ is the weighting factor in each equation and the sum over the weights is included for normalization. This phase uncertainty is used as described above (Eq. 3) to determine the amount of information retrieved from the measurement of that phase. To obtain the fidelity of information retrieval across the entire wave packet, we add the information capacity of each state so that the total information content is

$$I = \sum_k i_k = \sum_k \log \frac{2\pi}{\Delta\phi_k}. \qquad (5)$$

We perform our experiment with a Rydberg wave packet excited into np states with $n = 27 \ldots 32$. A correlation measurement on this wave packet allowed us to

determine $\Delta\phi = 7^\circ$ ($i = 5.7$ bits) for the $31p$ state. Fig.1 shows a correlation measurement when an HCP is applied at $T_1 = 5$ ps such that the correlation vanishes for all pairs of states involving $31p$. At this particular delay, population in state $31p$ has been substantially transferred to other states, both $\ell = 1$ and $\ell \neq 1$. Much of the phase information that was initially stored in the $31p$ state is now inaccessible to our measurement technique, which is only sensitive to the p-states. The phase uncertainty in the $31p$ state is 32° ($i = 3.5$ bits) from this measurement.

In the absence of a half-cycle pulse, the information capacity of the wave packet as determined using the correlation technique (Eq.3) is 35 bits. Following an HCP, for the data illustrated in Fig.1, this quantity becomes 29 bits. It can be seen from Fig. 1 that a large part of the phase information associated with $31p$ is lost.

When we apply a second HCP at a delay T_2 following the first HCP, it causes a redistribution of the states into the p-states depopulated by the first HCP in a coherent manner and we can once again measure the phase information in the previously missing state (see Fig. 2). The effect of the second HCP is to recover the information hidden by the first HCP. The uncertainty in the phase of the $31p$ state is reduced by nearly a factor of three from 32° to 12° ($i = 4.9$ bits).

The timing of the second HCP affects not only the final phase and amplitude of the depopulated state but also those of the other p-states. The second HCP will in general tend to depopulate those other p-states as well. At the delay shown in Fig.2, this produces a slight increase in the uncertainties of the phases of the other p-states (corresponding to less efficient information retrieval); this counteracts the information increase seen in $31p$. The total information content (28 bits) is nearly unchanged for this delay of the second HCP. The improvement that we observe is in the increase in information recovered from the $31p$ state.

We have also performed a separate experiment to determine whether the population transferred into the $31p$ state is due to p-state redistribution alone or if it is also the result of population transfer from non-p states as we expected. We isolate the effects of neighboring $\ell \neq 1$ states on the recovery of correlations by exciting a two-state wave packet where the two excited states are energetically distant, in our case, $27p$ and $32p$. Neither state has any p-state neighbors to which it is coupled by the weak HCP.

The effect of the HCP on the lower energy $27p$ state is minimal, while the same HCP causes significant transfer of $32p$ state amplitude into the neighboring $\ell \neq 1$ states, namely $31d$, $33s$, and $32s$. The final populations of the non-p states are small, relative to the $32p$ population. Note that what we measure as $32p$ population also contains $31d$ population; the states are nearly degenerate, and we do not resolve them using ramped field ionization.

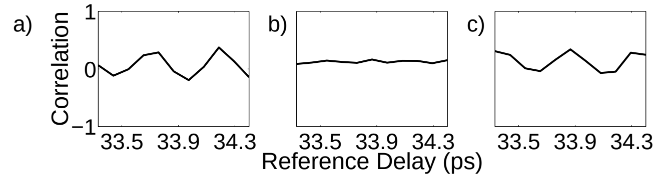

FIG. 3: Two-state wave packet correlations. a) In the absence of an HCP, the $27p$-$32p$ correlation is strong; the phase relationship is well-defined and easily read. b) $T_1 = 7$ ps. The operation of the HCP moves phase information into the $31d$ and $32s$ states, and that phase information becomes unreadable. c) $T_2 = 14.2$ ps. Application of a second HCP at a later delay recovers phase information in the $32p$ state.

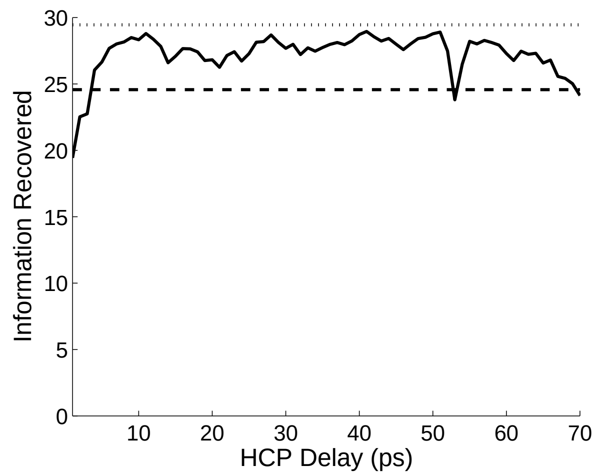

FIG. 4: Information recovered following second HCP. In the absence of any HCP, the information recovered is at the level indicated by the dotted line. An HCP arrives at a fixed delay $T_1 = 4.1$ ps after wave packet excitation, and the total phase information retrieved following this HCP kick is indicated by the dashed horizontal line. Following the first HCP, a second HCP kicks the wave packet at various delays, T_2. The information recovered following the second HCP (solid line) is shown as a function of the time difference between the first and second kicks.

In the two-state experiment, none of the population was transferred by the first HCP into other p states. Any population transferred back into $32p$ as the result of a second HCP must originate from a non-p state. Meanwhile, the effect of the HCP on the lower energy eigenstate ($27p$) is minimal. The correlation thus becomes a measure of the actual phase shift in $32p$.

Since only two p-states are initially excited, there is only a single correlation to be measured. In the absence of any HCP, the phase difference is well-defined (see Fig.3a). Upon application of a single HCP, the correlation amplitude between $32p$ and $27p$ vanishes (Fig.3b). The phase information is hidden because of depopulation of the $32p$ state in the presence of noise, as has

been previously established[10]. When an HCP kicks the wave packet a second time, the coherence between the $32p$ and $27p$ states is again observed (Fig.3c), as a result of transfer of coherent population from non-p states into the partially depopulated $32p$ state. This experiment proves that we can coherently transfer population from states that are accessible to our measurement to states that are inaccessible and then retrieve it at a later time.

We consider the robustness of the retrieved information as a function of the delay between the two HCP's. The first HCP is applied at $T_1 = 4.1$ ps such that it significantly depopulates one of the states in our wave packet. In Fig.4, the information capacity of the wave packet after a single HCP is represented by the dashed line. With the delay of the first HCP fixed, a second HCP arrives at various delays and the total information content following the two HCPs' is plotted as a function of this delay (solid line). It is seen that at nearly all delays the second HCP can reliably recover the information hidden by the first HCP.

The fundamental idea involved in the information hiding and retrieval scheme is that of transferring state amplitudes into different subspaces (those with $\ell \neq 1$), and transferring them back into a particular manifold which we are able to measure experimentally. As a further goal, we seek to learn about the content of states in a larger Hilbert space than we can measure directly. In the present work, both the excitation of the larger Hilbert space and its probe were half-cycle pulse operators. More generally, we find that we have a system which spans a large state space, of which we are only able to directly measure a small fraction. Such a scenario need not be limited to Rydberg atoms, but might include molecules and other multi-level systems.

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